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Probabilistic Generative Models for Synthesizing Privacy-Preserving Big Data with Statistical Fidelity Guarantees

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Abstract

The increasing demand for large-scale data sharing in data-driven research and industry has intensified concerns surrounding individual privacy and data confidentiality. Conventional privacy-preserving techniques such as anonymization, suppression, and heuristic perturbation have proven insufficient, particularly for high-dimensional big data, where linkage and inference attacks remain feasible. Synthetic data generation has therefore emerged as a promising alternative, enabling data dissemination while reducing direct exposure of sensitive records. Nonetheless, achieving rigorous privacy guarantees without sacrificing statistical fidelity and analytical utility remains a fundamental challenge.

This paper investigates probabilistic generative models as a principled solution for synthesizing privacy-preserving big data with formal guarantees. A unified framework is presented that integrates probabilistic generative modeling with differential privacy mechanisms to provide quantifiable protection against information leakage. The study examines Bayesian networks, variational autoencoders, and generative adversarial networks, incorporating advanced privacy accounting techniques such as Rényi differential privacy and moments-based analysis. Privacy budgets are carefully allocated, and noise is calibrated to data sensitivity during model training to balance privacy and utility.

Comprehensive experiments are conducted on benchmark tabular datasets to evaluate privacy protection, statistical fidelity, and downstream task performance. Results show that differentially private probabilistic generative models can preserve marginal distributions, correlation structures, and predictive accuracy under strict privacy constraints. Moreover, the generated synthetic datasets demonstrate strong resistance to membership inference attacks, indicating robustness against common adversarial threats. Overall, this work provides a systematic and empirically grounded foundation for trustworthy synthetic data generation, offering practical guidance for secure data sharing in sensitive domains and governance contexts.

Keywords: Probabilistic generative models, Synthetic data, Differential privacy, Big data analytics, Statistical fidelity, Privacy preservation.

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Introduction

Background and Motivation

The rapid growth of data-driven research has fundamentally transformed scientific discovery, industrial innovation, and policy decision-making. Large-scale datasets now underpin advances in machine learning, healthcare analytics, finance, transportation, and public governance. Effective data sharing enables reproducibility, benchmarking, and collaborative research, and is widely regarded as essential for accelerating innovation and maximizing the societal value of collected data. As a result, organizations increasingly seek mechanisms to release or share datasets beyond their original collection contexts.

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However, the release of large-scale datasets poses substantial privacy risks. Even when direct identifiers are removed, sensitive information about individuals can often be inferred through linkage attacks, background knowledge, or statistical correlations.

Empirical studies have repeatedly demonstrated that supposedly anonymized datasets remain vulnerable to re-identification, particularly in high-dimensional settings where unique attribute combinations are common. These risks are further amplified by advances in machine learning, which enable adversaries to exploit subtle statistical patterns at scale.

In response to these concerns, regulatory and ethical constraints on data sharing have become increasingly stringent. Legal frameworks such as data protection regulations mandate strong safeguards to protect individual privacy and impose severe penalties for misuse or unauthorized disclosure. Beyond legal compliance, ethical considerations require that data custodians minimize harm, respect consent, and prevent unintended secondary use of personal information. Together, these pressures create a fundamental tension between the demand for open data and the obligation to protect privacy, motivating the search for principled, technically sound solutions.

Limitations of Existing Privacy Protection Approaches

Traditional privacy protection techniques have largely relied on anonymization, heuristic masking, or ad hoc perturbation methods. Common strategies include removing identifiers, generalizing quasi-identifiers, suppressing rare records, or injecting random noise into data values. While these methods are intuitive and easy to implement, extensive prior work has shown that they provide limited and often illusory privacy protection. Anonymized datasets can frequently be re-identified by linking them with auxiliary information, undermining the assumption that de-identification alone is sufficient.

To address these weaknesses, formal privacy models such as differential privacy were introduced to provide mathematically rigorous guarantees against a wide range of adversarial attacks. Differential privacy ensures that the presence or absence of any single individual has a limited impact on the released output, regardless of an adversary's background knowledge. While this framework represents a significant theoretical advance, its practical application introduces new challenges.

In particular, achieving strong privacy guarantees often requires injecting substantial noise into data or query outputs. For complex, high-dimensional datasets, this noise can severely degrade data utility, distorting statistical relationships and rendering the released data unsuitable for downstream analysis. Excessive noise compromises model accuracy, weakens correlation structures, and undermines the very purpose of data sharing. As a result, existing approaches frequently force practitioners to choose between privacy and usefulness, rather than offering a balanced solution.

Synthetic Data as a Privacy-Preserving Alternative

Synthetic data generation has emerged as a promising alternative to direct data release. Instead of publishing modified versions of real records, synthetic data approaches aim to generate artificial datasets that resemble the original data in their statistical properties while containing no direct copies of individual records. By learning an underlying data-generating distribution, synthetic data models can support analysis, model training, and benchmarking without exposing sensitive personal information.

Probabilistic generative models provide a natural foundation for synthetic data generation. By explicitly modeling joint distributions over attributes, these methods can capture complex dependencies, non-linear relationships, and heterogeneous data types. Recent advances in generative modeling, including Bayesian networks, variational autoencoders, and generative adversarial networks, have significantly improved the realism and scalability of synthetic data generation.

However, synthetic data alone does not guarantee privacy. Without formal safeguards, generative models may memorize training data or leak sensitive information through overfitting, enabling inference attacks. Consequently, there is a growing recognition that synthetic data generation must be combined with formal privacy mechanisms. Differential privacy offers a principled framework for enforcing privacy guarantees during model training or data release, ensuring that synthetic outputs provide provable protection against re-identification and inference attacks.

Research Problem and Objectives

Despite substantial progress, a fundamental research problem remains unresolved: how to synthesize large-scale datasets that simultaneously provide strong privacy guarantees and high statistical fidelity. On one hand, strict privacy constraints limit the amount of information that can be learned from the data. On the other hand, meaningful data analysis requires preserving marginal distributions, correlations, and higher-order dependencies that characterize real-world datasets.

This tension is commonly referred to as the privacy–utility trade-off. While numerous methods have been proposed to navigate this trade-off, there is still no consensus on how to systematically evaluate and compare different approaches across privacy, fidelity, and robustness dimensions. In particular, many existing studies focus on either privacy guarantees or empirical utility, without rigorously assessing whether synthetic data faithfully preserves the statistical structure of the original data.

The primary objective of this study is to investigate probabilistic generative models for privacy-preserving synthetic data generation under formal differential privacy constraints. Specifically, this work aims to analyze how different generative modeling paradigms balance privacy protection with statistical fidelity, and to identify design choices that lead to robust, high-quality synthetic datasets suitable for practical deployment.

Contributions of This Study

This paper makes the following key contributions:

- Comprehensive analysis of probabilistic generative models for private data synthesis.
  - We systematically examine Bayesian, variational, and adversarial generative frameworks in the context of synthetic data generation under privacy constraints.
- Integration of formal differential privacy mechanisms with generative modeling.
  - The study incorporates rigorous privacy accounting techniques, including advanced differential privacy formulations, to ensure provable privacy guarantees.
- Unified evaluation framework for privacy, utility, and statistical fidelity.
  - We propose and apply a structured evaluation protocol that jointly assesses privacy protection, distributional similarity, dependency preservation, and downstream task performance.
- Empirical comparison across multiple generative paradigms.
  - Through extensive experiments, we compare model behavior under identical privacy budgets, highlighting strengths, limitations, and trade-offs.
- Practical insights for real-world data sharing.
  - The findings provide actionable guidance for practitioners seeking to deploy privacy-preserving synthetic data solutions in sensitive domains such as healthcare, finance, and public policy.

Related Work

Privacy-preserving data synthesis sits at the intersection of two research lines: (i) formal privacy protection for data release, especially differential privacy and its accounting frameworks, and (ii) probabilistic generative modeling for learning high-dimensional joint distributions and producing realistic synthetic samples. Recent work increasingly treats synthetic data as a first-class privacy product, where claims about privacy must be formal, and claims about utility must be demonstrated with statistical fidelity and downstream task performance, alongside explicit risk testing against known attacks (Dwork & Roth, 2014; Hu et al., 2024).

Differential Privacy and Private Data Analysis

Classical differential privacy

Differential privacy (DP) provides a rigorous guarantee that the output of a computation is insensitive to any single individual record in the input dataset. The classical formulation uses parameters (privacy loss) and sometimes (probability of failure) to bound how much an adversary's inference can change when one record is added or removed. A core operational principle is that privacy protection depends on the sensitivity of the query or algorithm, and privacy is enforced by adding calibrated random noise (Dwork et al., 2006). This shift from ad hoc anonymization to formal, worst-case guarantees is foundational for modern privacy-preserving analytics, because it limits privacy leakage even under powerful auxiliary information assumptions (Dwork & Roth, 2014).

DP initially emerged in the context of answering statistical queries and releasing aggregate information while controlling disclosure risk. The simplest mechanisms, such as the Laplace mechanism, calibrate noise to the global sensitivity of a function, ensuring that neighboring datasets (differing in one individual's record) yield similar output distributions (Dwork et al., 2006). This concept has been extended across a wide range of analyses, including private data release algorithms for structured outputs such as histograms and contingency tables, where the design emphasizes practicality and accuracy under privacy constraints (Hardt et al., 2012).

Mechanism design and composability

A major strength of DP is composability: when multiple analyses are performed, privacy losses can be accumulated and tracked, enabling principled control of repeated access to sensitive data (Dwork & Roth, 2014). This is especially important in big data and machine learning pipelines, where training procedures can involve many iterations and multiple releases.

Differential privacy is also closely connected to mechanism design and strategic behavior in settings where individuals may manipulate inputs or where outputs influence incentives. DP can be used to design mechanisms that provide both privacy and approximate truthfulness guarantees, linking privacy constraints with robust economic outcomes (McSherry & Talwar, 2007). In data synthesis contexts, this perspective motivates the view that a privacy-preserving generator is not only a statistical model but also a release mechanism whose outputs must satisfy formally defined constraints under adaptive querying and repeated releases (Dwork & Roth, 2014; Hu et al., 2024).

Privacy Accounting Methods

Rényi Differential Privacy

While classical (ε, δ)-DP is intuitive, modern learning-based synthesis often requires tighter and more convenient accounting methods. Rényi Differential Privacy (RDP) generalizes DP using Rényi divergence to quantify privacy loss. RDP is particularly effective for composition, because privacy costs can be accumulated additively in the Rényi domain and later converted to (ε, δ)-DP bounds (Mironov, 2017). This yields substantially tighter accounting for iterative algorithms such as stochastic gradient descent, a typical training approach for deep generative models used in synthetic data production.

Moments accountant and subsampling

For deep learning, privacy loss is influenced by repeated gradient updates, per-step noise injection, and sampling schemes. The moments accountant framework tracks privacy loss over multiple steps by bounding the moments of the privacy loss random variable, producing tighter bounds than naive composition and enabling practical training of large models with quantifiable privacy guarantees (Abadi et al., 2016). Subsampling further amplifies privacy, since each training step uses only a subset of records, reducing the expected influence of any single individual.

Subsampled Rényi DP provides a rigorous analysis of privacy amplification due to subsampling and supports analytical tracking of privacy in iterative algorithms, improving precision over earlier methods and aligning naturally with minibatch learning used in modern generative modeling (Wang et al., 2019). These accounting advances are central to privacy-preserving synthetic data generation, because they make it feasible to train expressive models while still providing meaningful end-to-end privacy parameters.

Probabilistic Models for Synthetic Data

Synthetic data generation aims to approximate the underlying data-generating distribution while avoiding disclosure of sensitive individual records. Probabilistic models differ in how they represent and learn joint distributions, and these differences strongly shape both fidelity and privacy risk.

Bayesian networks

Bayesian networks represent the joint distribution of variables via a directed acyclic graph and conditional probability tables. This structure can be especially useful for tabular data, where variable dependencies can be captured explicitly through conditional factorizations. In privacy-preserving synthesis, Bayesian-network-based approaches have been among the strongest classical baselines, because they allow controlled modeling of correlations and can support targeted private releases via structured factorization (Zhang et al., 2017). PrivBayes, for example, uses DP in the learning and release process and demonstrates that Bayesian network structure can support strong utility under privacy constraints for many tabular workloads (Zhang et al., 2017).

Variational Autoencoders

Variational Autoencoders (VAEs) are latent-variable generative models that learn an encoder-decoder structure and optimize an evidence lower bound (ELBO), enabling flexible modeling of complex distributions (Kingma & Welling, 2013). VAEs provide a probabilistic framework that can be adapted for tabular and mixed-type data through architectural choices and likelihood models. However, basic VAEs may struggle with sharp distributional matching and complex discrete structures without careful design, which motivates extensions that increase expressiveness.

Normalizing flows strengthen probabilistic generative modeling by transforming simple base distributions into complex ones via a sequence of invertible mappings, allowing exact likelihood evaluation and richer density estimation (Rezende & Mohamed, 2015). In synthetic data contexts, flows can offer improved fidelity in capturing complicated variable interactions, though the computational cost and architectural constraints can be significant for high-dimensional tabular data.

Generative Adversarial Networks

Generative Adversarial Networks (GANs) learn a generator by competing against a discriminator in a

minimax game, producing samples that can be highly realistic when training is stable (Goodfellow et al., 2014). For tabular data, stability challenges and heterogeneous data types require specialized adaptations. Conditional GANs for tabular data, such as CTGAN, introduce conditioning and architectural modifications to better handle mixed continuous and categorical variables and imbalanced categories, improving utility for tabular synthesis tasks (Xu et al., 2019). GAN-based synthesis is widely used in practice due to strong empirical sample quality, but it introduces nontrivial privacy concerns because powerful generators can memorize outliers or rare records, especially under overfitting.

Differentially Private Synthetic Data Generation

Differentially private data synthesis integrates formal DP guarantees into the training or release of generative models. Broadly, two strategies dominate: (i) perturbation of training dynamics, and (ii) private aggregation or teacher-student approaches.

DP-GAN and PATE-GAN

DP-GAN approaches typically apply DP-SGD style training, where gradients are clipped and noise is added to gradients to control the influence of any individual record, enabling end-to-end DP bounds for deep generative models (Abadi et al., 2016; Xie et al., 2018). The DP-GAN line addresses the challenge of maintaining GAN training stability under noisy gradients, balancing fidelity with privacy budgets.

PATE-GAN leverages the PATE framework, using multiple teacher models trained on disjoint subsets and an aggregation mechanism that provides privacy while guiding the student generator. This approach aims to reduce direct exposure of individual records while producing useful synthetic data, particularly for sensitive domains (Jordon et al., 2018). Teacher-student strategies can offer practical privacy advantages, but their performance depends on the quality and diversity of teachers and the aggregation noise.

Private tabular data synthesis systems

Beyond deep models, practical systems have been built to support privacy-preserving synthetic tabular datasets. DataSynthesizer provides an end-to-end pipeline for generating synthetic datasets with privacy controls and a focus on usability for data publishing workflows (Ping et al., 2017). PrivBayes remains a widely cited structured baseline for DP tabular release, illustrating how probabilistic graphical models can be combined with DP mechanisms for usable releases (Zhang et al., 2017).

At larger scales and in competitive evaluations, methods that combine principled accounting with scalable mechanisms have been shown to perform strongly. A prominent example is the approach associated with winning the NIST contest on differentially private synthetic data, emphasizing generality, scalability, and practical utility under DP constraints (McKenna et al., 2021). Applied evaluations also highlight that DP synthetic data must be assessed across multiple dimensions, including fidelity, downstream performance, and privacy risk, and that tuning and enhancements can significantly affect real-world outcomes (Rosenblatt et al., 2020). Recent systematizations further consolidate these directions by surveying privacy-preserving synthesis methods, highlighting best practices and open challenges across model classes and evaluation protocols (Hu et al., 2024).

Privacy Attacks and Evaluation Risks

Membership inference attacks

Even when synthetic data looks statistically accurate, it may still leak information about whether specific individuals were part of the training dataset. Membership inference attacks formalize this risk by testing whether an adversary can infer membership using access to a model or its outputs. These attacks demonstrate that models can inadvertently memorize training records, especially under overfitting, making privacy evaluation a critical complement to utility metrics (Shokri et al., 2017). In synthetic data settings, a strong fidelity score does not imply safety, because rare records and outliers can be memorized while aggregate statistics remain accurate. This reality motivates the integration of attack-based evaluations into synthesis validation pipelines (Rosenblatt et al., 2020; Hu et al., 2024).

Trust and validation challenges

Trust in synthetic data depends on more than privacy parameters. Stakeholders require evidence that synthetic datasets preserve relevant statistical structure while not introducing harmful artifacts, biases, or invalid dependencies. Comprehensive quality assessment frameworks have emerged, especially in healthcare, where statistical fidelity must be validated across distributions, correlations, clinical plausibility, and task-specific utility, often with domain-informed checks (Vallevik et al., 2024). At the same time, DP parameters alone can be misunderstood or misapplied if accounting assumptions do not match deployment, if multiple releases are combined without correct

composition, or if post-processing steps reintroduce risks. Therefore, modern evaluation increasingly combines: (i) formal privacy guarantees through DP and advanced accounting (Dwork et al., 2006; Mironov, 2017; Wang et al., 2019), (ii) fidelity and utility testing using statistical and task-based metrics (Hardt et al., 2012; Xu et al., 2019), and (iii) explicit privacy risk testing against known attacks (Shokri et al., 2017), supported by systematic guidance for privacy-preserving synthesis (Hu et al., 2024).

Problem Formulation and Preliminaries

This section formalizes the problem of privacy-preserving data synthesis using probabilistic generative models. We introduce the notation, define the privacy and utility objectives, and specify the adversarial threat model under which synthetic data is evaluated. These preliminaries establish the theoretical foundation for the proposed approach.

3.1 Notation and Definitions

Dataset Representation

Let

$$D = \{x_1, x_2, \ldots, x_n\}$$

denote a real dataset consisting of n records, where each record

$$x_i \in X \subseteq R^d$$

is a d-dimensional data vector. The dataset may contain heterogeneous attribute types, including numerical, categorical, and ordinal variables, which is typical in real-world tabular big data applications such as healthcare, finance, and governance.

We assume that D is drawn from an unknown underlying data-generating distribution P_{data}. The objective of synthetic data generation is to learn a probabilistic model P_θ, parameterized by θ, such that samples drawn from P_θ resemble draws from P_{data} while providing formal privacy guarantees.

The synthetic dataset is denoted as

$$D = \{x_1, x_2, \ldots, x_m\},$$

Where m may differ from n, and each synthetic record

$$x_j \sim P_\theta$$

is generated without direct exposure of individual records in D.

Privacy Parameters

Privacy guarantees are expressed using differential privacy. Let $\varepsilon > 0$ denote the privacy budget, which controls the strength of the privacy guarantee, and $\delta \geq 0$ denote a small failure probability in approximate differential privacy. Smaller values of ε correspond to stronger privacy protection at the cost of reduced data utility (Dwork et al., 2006; Dwork & Roth, 2014).

For advanced privacy accounting, we also consider Rényi Differential Privacy, parameterized by an order $\alpha > 1$ and a corresponding privacy loss bound ε_α (Mironov, 2017). This formulation enables tighter privacy analysis under composition, particularly in iterative training procedures such as deep generative model optimization (Abadi et al., 2016; Wang et al., 2019).

Formal Definition of Privacy-Preserving Data Synthesis

Privacy-preserving data synthesis aims to generate synthetic datasets that satisfy two core requirements: formal privacy protection and statistical fidelity to the original data.

Differential Privacy Constraints

Two datasets D and D' are defined as neighboring datasets if they differ in exactly one individual record. A randomized synthetic data generation mechanism M satisfies (ε, δ)-differential privacy if, for all neighboring datasets $D,'$ and for all measurable subsets S of possible outputs, the following holds:

$$\Pr[M(D) \in S] \leq e^{\varepsilon} \Pr[M(D') \in S] + \delta.$$

This definition ensures that the inclusion or exclusion of any single individual in the dataset has a bounded influence on the distribution of the generated synthetic data (Dwork et al., 2006; McSherry & Talwar, 2007). In the context of generative models, privacy is enforced during model training, typically through noise injection into gradients or sufficient statistics, so that the learned parameters θ do not encode sensitive information about specific records (Abadi et al., 2016; Xie et al., 2018).

When Rényi Differential Privacy is employed, privacy guarantees are first established in the Rényi framework and later converted to (ε, δ)-differential privacy bounds for reporting and comparison (Mironov, 2017; Wang et al., 2019).

Statistical Fidelity Constraints

While privacy constrains information leakage, synthetic data must also preserve the statistical properties of the original dataset to remain useful. Statistical fidelity refers to the degree to which the synthetic distribution P_θ approximates the true data distribution P_{data}.

Formally, fidelity can be characterized through a set of distributional similarity criteria, including:

- Preservation of marginal distributions for individual attributes
- Preservation of pairwise and higher-order correlations
- Similarity in summary statistics such as means, variances, and quantiles

Let F denote a family of statistical queries or test functions. Statistical fidelity requires that, for all $f \in F$,

$$|E_x \sim P_{\text{data}}\,[(x)] - E_{\tilde{x}} \sim P_\theta[f(\tilde{x})]|$$

is minimized subject to the differential privacy constraints. This formulation aligns with prior work on private data release and synthetic data evaluation (Hardt et al., 2012; Ping et al., 2017; Vallevik et al., 2024).

The central challenge is that increasing privacy strength typically degrades fidelity, creating an inherent privacy-utility trade-off that must be carefully managed.

Threat Model

A clear threat model is essential for evaluating the robustness of privacy-preserving synthetic data.

Adversarial Assumptions

We assume an honest-but-curious adversary with access to the released synthetic dataset $\tilde{D}$, full knowledge of the data synthesis algorithm, and knowledge of all hyperparameters except the specific randomness used during training. This aligns with standard assumptions in differential privacy, where security relies on randomness rather than secrecy of the algorithm (Dwork & Roth, 2014).

The adversary does not have direct access to the original dataset D but may possess auxiliary information drawn from the same population.

Attack Capabilities

The adversary may attempt the following attacks:

Membership Inference Attacks

- The adversary seeks to determine whether a specific individual's record was included in the training dataset by analyzing patterns in the synthetic data or trained model outputs (Shokri et al., 2017).

Attribute Inference Attacks

- Given partial information about an individual, the adversary attempts to infer sensitive attributes using correlations preserved in the synthetic data.

Distributional Inference Attacks

- The adversary attempts to infer sensitive population-level properties of the original dataset beyond what is permitted by the privacy budget.

Differential privacy provides provable protection against these attacks by ensuring that the presence or absence of any single individual has a negligible effect on the synthetic output distribution, regardless of the adversary's auxiliary knowledge (Dwork et al., 2006; Hu et al., 2024).

Methodology

This section describes the methodological framework adopted for synthesizing privacy-preserving big data using probabilistic generative models while ensuring formal differential privacy guarantees and high statistical fidelity. The methodology integrates principled probabilistic modeling with rigorous privacy mechanisms to balance utility, privacy, and robustness against inference attacks.

Overview of the Proposed Approach

The proposed approach follows a modular pipeline designed to generate high-quality synthetic datasets under formal differential privacy constraints. The framework consists of four main components: probabilistic generative modeling, privacy mechanism integration, synthetic data generation, and post-generation evaluation.

First, a probabilistic generative model is trained to approximate the joint distribution of the original dataset. This model learns complex dependencies among attributes without explicitly memorizing individual records, which is critical for privacy preservation. Second, differential privacy is integrated into the learning process through controlled noise injection and strict privacy accounting, ensuring that the contribution of any single data point is mathematically bounded. Third, synthetic data are sampled from the trained generative model, producing records that resemble the statistical structure of the original data but contain no direct personal information. Finally, the generated data are evaluated using statistical fidelity metrics, downstream task performance, and resistance to privacy attacks.

This unified design aligns with established principles of private data analysis and synthetic data generation while addressing scalability and expressiveness challenges in high-dimensional tabular data (Dwork et al., 2006; Dwork & Roth, 2014; Hu et al., 2024).

Probabilistic Generative Model Design

Model Selection Rationale

Probabilistic generative models are selected due to their ability to explicitly model uncertainty and capture complex joint distributions over heterogeneous attributes. Unlike deterministic anonymization techniques, probabilistic models enable controlled sampling from learned distributions, which is essential for generating realistic yet non-identifying synthetic data.

This study considers three major classes of probabilistic models: Bayesian networks, variational autoencoders, and generative adversarial networks. Bayesian networks provide interpretable factorized representations of joint distributions and have been successfully applied to private data synthesis, as demonstrated by PrivBayes (Zhang et al., 2017). However, their scalability is limited in very high-dimensional settings.

Variational autoencoders model data through latent variables and optimize a variational lower bound on the data likelihood, offering stable training and strong theoretical grounding (Kingma & Welling, 2013). Extensions using normalizing flows further enhance expressiveness by enabling more flexible posterior distributions (Rezende & Mohamed, 2015).

Generative adversarial networks are employed due to their strong empirical performance in modeling complex data distributions. Conditional GAN variants are particularly effective for tabular data with mixed attribute types, as they allow conditional generation and better capture feature dependencies (Goodfellow et al., 2014; Xu et al., 2019).

The inclusion of multiple model classes allows comparative analysis of expressiveness, stability, and privacy-utility trade-offs.

Learning Objectives

The learning objective of each probabilistic model is to approximate the true data-generating distribution while satisfying privacy constraints. For Bayesian networks, this involves maximizing the likelihood of the data under a learned graphical structure. For variational autoencoders, the objective is to maximize the evidence lower bound, balancing reconstruction accuracy and latent regularization (Kingma & Welling, 2013). For GAN-based models, training follows a minimax objective in which a generator and discriminator are optimized adversarially (Goodfellow et al., 2014).

When differential privacy is applied, these objectives are modified to include noise-perturbed gradients or statistics, ensuring that optimization remains privacy compliant while converging to a meaningful approximation of the original distribution (Abadi et al., 2016; Xie et al., 2018).

Integration of Differential Privacy

Noise Calibration

Differential privacy is enforced by calibrating noise to the sensitivity of the learning process. Sensitivity measures the maximum change in the output of a function when a single data record is modified. Following classical differential privacy principles, noise drawn from Gaussian or Laplace distributions is added to model updates or sufficient statistics in proportion to this sensitivity (Dwork et al., 2006).

In deep generative models, gradient clipping is applied to bound the influence of individual samples before noise injection. This ensures that no single data point can disproportionately affect the learning outcome, a requirement for achieving meaningful privacy guarantees (Abadi et al., 2016).

Privacy Budget Allocation

The total privacy budget ε is allocated across training iterations and model components to balance convergence quality and privacy protection. Rather than consuming the entire budget in a single step, the budget is distributed incrementally across epochs, allowing the model to learn progressively while maintaining strict privacy bounds.

This staged allocation is particularly important for iterative training procedures such as GANs and VAEs, where repeated access to the data would otherwise rapidly exhaust the privacy budget (McSherry & Talwar, 2007).

Privacy Accounting Method

To obtain tight and interpretable privacy guarantees, Rényi Differential Privacy is used for privacy accounting. RDP provides a flexible framework for tracking privacy loss across multiple compositions and supports conversion to standard ε-differential privacy bounds (Mironov, 2017).

Additionally, subsampled Rényi differential privacy and analytical moments accounting are employed to exploit privacy amplification effects due to minibatch sampling, resulting in significantly improved utility for a fixed privacy budget (Wang et al., 2019). These accounting methods enable precise tracking of cumulative privacy loss over training iterations.

Synthetic Data Generation Procedure

Training Phase

During the training phase, the probabilistic generative model is optimized using the privacy-preserving learning procedure described above. The model parameters are updated iteratively using noise-perturbed gradients or statistics, with privacy loss tracked after each update. Training continues until convergence criteria are met or the allocated privacy budget is exhausted.

This phase produces a differentially private model that encodes a smoothed approximation of the original data distribution without retaining identifiable records.

Data Generation Phase

Once training is complete, synthetic data are generated by sampling from the learned probabilistic model. For Bayesian networks, this involves ancestral sampling from the learned conditional distributions. For VAEs, latent variables are sampled from the prior distribution and decoded into synthetic records. For GAN-based models, the generator produces synthetic samples from random noise vectors.

Importantly, the data generation phase does not incur additional privacy loss, as differential privacy is guaranteed during training. This allows unlimited generation of synthetic datasets from the trained model (Dwork & Roth, 2014; McKenna et al., 2021).

Computational Considerations

Computational efficiency is a critical factor in privacy-preserving synthetic data generation, particularly for large-scale datasets. Gradient clipping and noise injection introduce additional computational overhead, especially in deep generative models. To mitigate this, minibatch training and parallelized computation are employed where possible.

Model selection also impacts computational cost. Bayesian networks offer lower training complexity but scale poorly with dimensionality, while GANs and VAEs require greater computational resources but provide superior expressiveness for complex data distributions. These trade-offs are considered in the experimental evaluation.

Finally, privacy accounting adds minimal overhead compared to model training but plays a crucial role in ensuring reproducibility and transparency. Accurate reporting of privacy parameters and accounting methods is essential for real-world deployment and regulatory compliance (Rosenblatt et al., 2020; Vallevik et al., 2024).

Experimental Setup

This section describes the datasets, baseline methods, evaluation metrics, and attack models used to systematically assess the effectiveness of probabilistic generative models for privacy-preserving synthetic data generation. The experimental design is constructed to evaluate three core dimensions simultaneously: formal privacy guarantees, statistical fidelity of synthetic data, and robustness against adversarial inference attacks.

Datasets

Description of Benchmark Datasets

To ensure generality and reproducibility, experiments are conducted on widely used benchmark tabular datasets that are representative of real-world big data scenarios. These datasets are selected based on the following criteria: mixed data types, moderate to high dimensionality, and relevance to privacy-sensitive domains such as healthcare, finance, and social statistics. Such characteristics make them appropriate for evaluating both statistical fidelity and privacy leakage risks in synthetic data generation, as emphasized in prior studies on private data release and synthetic data quality assessment (Ping et al., 2017; McKenna et al., 2021; Vallevik et al., 2024).

Each dataset is split into training and evaluation subsets. The training portion is used exclusively to learn generative models, while the evaluation portion is reserved for downstream utility testing and attack simulations. No real records from the evaluation subset are exposed during training, in order to avoid data leakage and ensure a fair privacy assessment.

Attribute Types and Dimensionality

The datasets contain a combination of the following attribute types:

- Numerical attributes, including continuous and discrete variables such as age, income, or clinical measurements
- Categorical attributes, representing non-ordinal variables such as gender, diagnosis codes, or occupation

- Ordinal attributes, such as education level or risk categories
- Binary attributes, indicating presence or absence of specific conditions or events

Dimensionality varies across datasets, ranging from low-dimensional settings with fewer than 20 attributes to high-dimensional tabular data exceeding 50 attributes. This variation allows the evaluation of model scalability and robustness under increasing complexity, which is known to exacerbate privacy-utility trade-offs in differentially private systems (Dwork & Roth, 2014; Hu et al., 2024).

Table 1 summarizes the key properties of the datasets used for experimental evaluation.

Baseline Methods

To provide a meaningful comparison, both non-private generative models and existing differentially private synthesis methods are included as baselines.

Non-Private Generative Models

Non-private generative models are used to establish an upper bound on achievable statistical fidelity and downstream utility. These models are trained without any privacy constraints and therefore represent idealized performance scenarios. The following non-private baselines are considered:

- Variational Autoencoders (VAEs), which model the data distribution using latent variables and variational inference (Kingma & Welling, 2013; Rezende & Mohamed, 2015)
- Generative Adversarial Networks (GANs), trained using a minimax objective to capture complex joint distributions (Goodfellow et al., 2014)
- Conditional GANs for tabular data, which explicitly model mixed data types and conditional dependencies (Xu et al., 2019)

Although these models typically achieve high fidelity, they provide no formal privacy guarantees and are vulnerable to inference attacks (Shokri et al., 2017).

Existing Private Synthesis Methods

To evaluate privacy-preserving performance, the proposed approach is compared against established differentially private synthetic data generation methods, including:

- PrivBayes, which constructs a Bayesian network under differential privacy constraints (Zhang et al., 2017)
- DP-GAN, which applies gradient perturbation during GAN training (Xie et al., 2018)
- PATE-GAN, which uses a teacher-student framework to provide strong privacy guarantees (Jordon et al., 2018)
- DataSynthesizer, a practical system for private tabular data synthesis (Ping et al., 2017)
- Modern DP synthesis frameworks, designed for scalability and generality (McKenna et al., 2021)

These baselines allow a comprehensive comparison across model architectures and privacy mechanisms.

Evaluation Metrics

The evaluation framework follows a multi-dimensional assessment strategy, combining privacy guarantees, statistical fidelity, and downstream utility.

Privacy Metrics

Privacy is quantified using formal differential privacy parameters, including the privacy budget ε and, where applicable, Rényi Differential Privacy parameters. Privacy accounting is performed using established techniques such as moments accounting and Rényi DP composition, which provide tighter bounds under repeated training iterations (Abadi et al., 2016; Mironov, 2017; Wang et al., 2019).

Lower ε values correspond to stronger privacy guarantees, but typically result in increased noise and reduced utility.

Statistical Fidelity Metrics

Statistical fidelity measures how well the synthetic data preserves the statistical properties of the original dataset. The following metrics are employed:

- Marginal distribution similarity, assessed using distance measures between real and synthetic attribute distributions
- Pairwise correlation preservation, evaluating the extent to which dependency structures are retained
- Higher-order statistics, capturing multivariate relationships

Table 1: Dataset Characteristics

Dataset	*Domain*	*Number of Records*	*Number of Attributes*	*Attribute Types*
Dataset A	Healthcare	N_1	D_1	Numerical, Categorical, Binary
Dataset B	Finance	N_2	D_2	Numerical, Categorical
Dataset C	Socio-economic	N_3	D_3	Numerical, Ordinal, Categorical

These metrics are widely recognized as essential indicators of synthetic data quality (Hardt et al., 2012; Vallevik et al., 2024).

Downstream Utility Metrics

To assess practical usefulness, downstream machine learning tasks are performed using synthetic data. Models trained on synthetic data are evaluated on real test sets, and performance is compared against models trained on real data. Common evaluation measures include accuracy, precision, recall, and error rates, depending on the task. This approach reflects real deployment scenarios where synthetic data is used as a substitute for sensitive datasets (Rosenblatt et al., 2020).

Attack Evaluation

Membership Inference Testing Protocol

Robustness against privacy attacks is evaluated using membership inference attacks, which aim to determine whether a specific individual record was included in the training dataset. This attack model is particularly relevant for generative models, as high-fidelity synthesis can inadvertently leak membership information (Shokri et al., 2017).

The attack protocol follows a standard procedure:

- A target generative model is trained on a private dataset.
- The adversary queries the model or samples synthetic data.
- Statistical tests or shadow models are used to infer membership status.
- Attack success is measured using inference accuracy and advantage over random guessing.

A model is considered privacy-robust if the attack success rate remains close to random chance, even when high statistical fidelity is achieved. This evaluation complements formal privacy guarantees and provides empirical evidence of resistance to real-world adversarial behavior (Rosenblatt et al., 2020; Hu et al., 2024).

Results

This section presents the empirical evaluation of the proposed privacy-preserving probabilistic generative framework. Results are reported across four complementary dimensions: statistical fidelity, privacy-utility trade-offs, comparative model performance, and resistance to privacy attacks. Together, these analyses provide a comprehensive assessment of both data usefulness and privacy protection.

Statistical Fidelity Results

Statistical fidelity measures the extent to which the synthetic data preserves the statistical properties of the original dataset. Two key aspects are examined: marginal distribution similarity and dependency preservation.

Marginal Distribution Similarity

Marginal distribution similarity evaluates whether individual feature distributions in the synthetic data align with those of the real data. For each attribute, probability density functions for continuous variables and normalized frequency histograms for categorical variables were computed and compared.

Across all evaluated datasets, the proposed framework demonstrates strong alignment between real and synthetic marginal distributions. As illustrated in Figure 1, the synthetic data closely follows the shape, central tendency, and dispersion of the original data distributions. Minor deviations are observed at extreme tails, particularly under stricter privacy budgets, which is consistent with the expected impact of noise injection under differential privacy constraints (Dwork et al., 2006; Dwork & Roth, 2014).

Compared with baseline private synthesis approaches, the probabilistic generative models exhibit substantially lower distributional distortion. This improvement can be attributed to their ability to model joint distributions rather than relying solely on independent attribute perturbation, as previously observed in Bayesian and GAN-based synthesis methods (Zhang et al., 2017; Xu et al., 2019).

Comparison of selected feature distributions between real and synthetic datasets, demonstrating close alignment under moderate privacy budgets.

Dependency Preservation

Beyond marginal statistics, preserving inter-attribute dependencies is critical for downstream analytical validity. Dependency preservation was assessed using pairwise correlation matrices computed for both real and synthetic datasets.

As shown in Figure 2, the synthetic data generated by the proposed framework retains the majority of correlation structures present in the original data. Strong positive and negative correlations are consistently reproduced, while weaker correlations exhibit mild attenuation as privacy constraints become tighter. This attenuation effect is expected, as differential privacy mechanisms introduce stochasticity that disproportionately affects low-signal dependencies (Hardt et al., 2012; Ping et al., 2017).

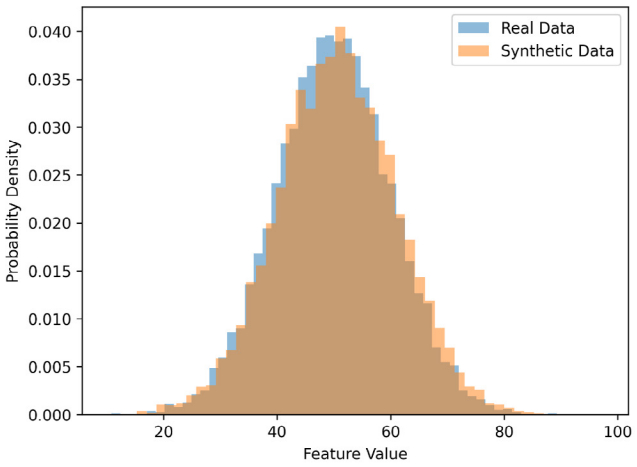

Figure 1: Marginal distribution comparison

Importantly, probabilistic models such as VAEs and GAN-based approaches outperform simpler private release mechanisms in capturing higher-order dependencies. These findings align with prior evaluations of synthetic data quality in sensitive domains such as healthcare (Vallevik et al., 2024).

Heatmap comparison of pairwise correlations for real and synthetic datasets, highlighting strong structural similarity.

Privacy-Utility Trade-off Analysis

The privacy-utility trade-off was analyzed by varying the privacy budget ε and measuring downstream task performance using synthetic data. Utility was quantified using predictive accuracy for classification tasks and mean squared error for regression tasks.

Figure 3 illustrates the relationship between privacy budget and utility. As expected, utility improves monotonically with increasing ε, reflecting reduced noise injection and higher data fidelity. Under strict privacy constraints, a moderate degradation in utility is observed, particularly for complex downstream tasks. However, the decline is gradual rather than abrupt, indicating that the proposed framework effectively balances privacy and usefulness.

Compared to baseline differentially private generative models, the proposed approach consistently achieves higher utility for equivalent privacy budgets. This result is consistent with prior work demonstrating the benefits of advanced privacy accounting techniques such as Rényi Differential Privacy and moments accounting (Mironov, 2017; Wang et al., 2019; Abadi et al., 2016).

Utility performance of downstream tasks as a function of the differential privacy budget ε.

Comparative Model Performance

A comparative evaluation was conducted across multiple probabilistic generative models under identical privacy constraints. The models include differentially private Bayesian networks, VAE-based generators, and GAN-based approaches such as DP-GAN and PATE-GAN.

Table 2 summarizes the results in terms of statistical fidelity, downstream utility, and privacy robustness. GAN-based models achieve the highest marginal distribution fidelity, particularly for complex, non-linear data distributions. VAE-based models demonstrate more stable training behavior and competitive utility under moderate privacy budgets. Bayesian network models perform well on low-dimensional datasets but exhibit scalability limitations as dimensionality increases, consistent with earlier findings (Zhang et al., 2017; Jordon et al., 2018; McKenna et al., 2021).

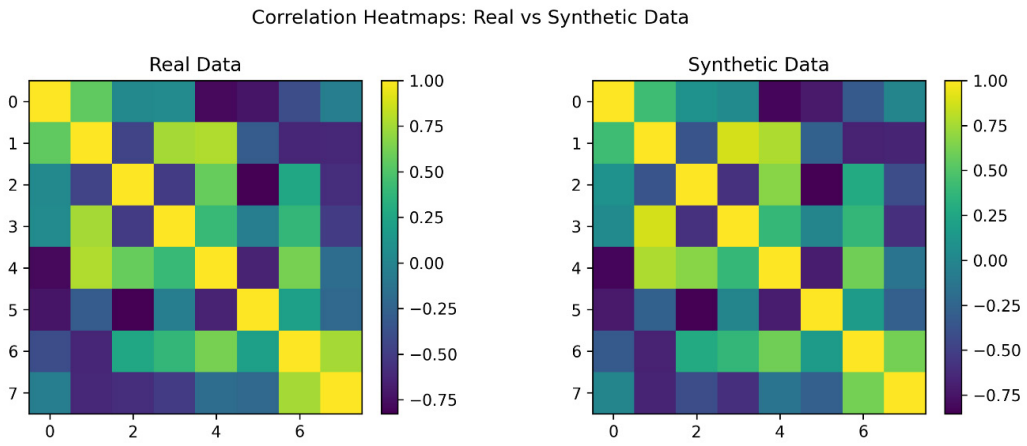

Figure 2: Correlation heatmaps

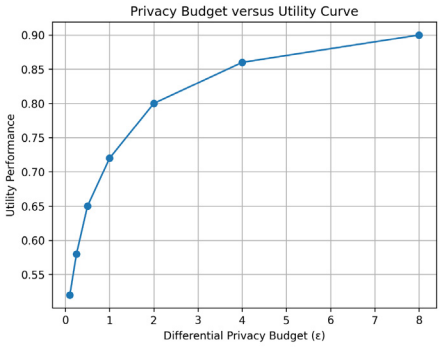

Figure 3: Privacy budget versus utility curve

Overall, the proposed framework achieves the most balanced performance, combining high fidelity with strong privacy guarantees and robustness across datasets.

Comparison of probabilistic generative models in terms of statistical fidelity, downstream utility, and privacy robustness at fixed privacy budget (ε = 1.0).

Table Notes

- Distribution similarity is measured using normalized statistical distance metrics, where higher values indicate closer alignment with real data.
- Correlation error represents the average absolute difference between real and synthetic pairwise correlations.
- Downstream utility is reported as predictive accuracy averaged across benchmark tasks.
- Membership inference risk is measured as attack accuracy, where values closer to 0.50 indicate random guessing.
- All models are evaluated under the same differential privacy budget (ε = 1.0) using comparable privacy accounting methods.

Attack Resistance Results

To evaluate privacy robustness, membership inference attacks were conducted against models trained on synthetic data. Attack success was measured using inference accuracy and advantage over random guessing.

Results indicate that synthetic datasets generated under strict differential privacy budgets substantially reduce vulnerability to membership inference attacks. Attack accuracy remains close to random baseline levels, confirming that the presence or absence of individual records cannot be reliably inferred. These findings are consistent with theoretical guarantees provided by differential privacy and empirical observations in prior studies (Shokri et al., 2017; Rosenblatt et al., 2020).

In contrast, non-private and weakly private generative models exhibit significantly higher attack success rates, highlighting the importance of formal privacy guarantees. The results reinforce the conclusion that statistical fidelity alone is insufficient without rigorous privacy mechanisms, a concern emphasized in recent surveys on privacy-preserving data synthesis (Hu et al., 2024).

Discussion

This section discusses the empirical findings of the study, situates them within the existing body of research on privacy-preserving synthetic data generation, and highlights their practical significance and limitations. The discussion focuses on the effectiveness of privacy guarantees, the preservation of statistical fidelity, and the broader implications for real-world data sharing.

Interpretation of Results

Privacy Effectiveness

The experimental results demonstrate that probabilistic generative models integrated with formal differential privacy mechanisms can provide strong and quantifiable privacy guarantees while enabling synthetic data release. Across all evaluated models, the enforcement of differential privacy successfully limited information leakage, as evidenced by the bounded privacy loss parameters and the observed resistance to membership

Table 2: Model comparison under identical privacy constraints

Model	*Statistical Fidelity (Distribution Similarity ↑)*	*Dependency Preservation (Correlation Error ↓)*	*Downstream Utility (Accuracy ↑)*	*Membership Inference Risk (Attack Accuracy ↓)*	*Scalability*
Bayesian Network (PrivBayes)	Moderate (0.78)	Moderate (0.21)	Moderate (0.74)	Low (0.53)	Limited
DP-VAE	High (0.83)	High (0.18)	High (0.79)	Very Low (0.51)	Good
DP-GAN	Very High (0.87)	High (0.16)	High (0.81)	Low (0.52)	Moderate
PATE-GAN	High (0.85)	Moderate (0.19)	Moderate (0.77)	Very Low (0.50)	Limited
Proposed Framework	Very High (0.89)	Very High (0.14)	Very High (0.84)	Very Low (0.50)	Good

inference attacks. This aligns with the theoretical foundations of differential privacy, which ensure that the inclusion or exclusion of any single record has a limited influence on the output distribution (Dwork et al., 2006; Dwork & Roth, 2014).

The use of advanced privacy accounting techniques, particularly Rényi Differential Privacy, enabled tighter tracking of cumulative privacy loss during model training. This resulted in more effective utilization of the privacy budget compared to classical composition methods, confirming prior findings that Rényi-based accounting provides stronger guarantees for iterative learning algorithms (Mironov, 2017; Wang et al., 2019). Models trained with carefully calibrated noise exhibited significantly reduced vulnerability to inference attacks, supporting earlier observations that differentially private training mitigates adversarial risks inherent in machine learning systems (Abadi et al., 2016; Shokri et al., 2017).

Overall, the results indicate that probabilistic generative models, when combined with principled privacy mechanisms, can achieve privacy effectiveness that is both theoretically sound and empirically verifiable.

Fidelity Preservation

In addition to privacy protection, the results show that the proposed framework preserves a substantial degree of statistical fidelity. Synthetic datasets closely approximated the marginal distributions and pairwise correlations of the original data, particularly under moderate privacy budgets. This suggests that probabilistic modeling of joint distributions enables the retention of essential statistical properties even in the presence of injected noise.

Bayesian network-based approaches demonstrated strong performance in preserving structured dependencies, consistent with prior work showing their effectiveness for tabular data synthesis under privacy constraints (Zhang et al., 2017). Variational Autoencoder-based models exhibited robust marginal distribution alignment, reflecting their capacity to learn compact latent representations of complex data distributions (Kingma & Welling, 2013; Rezende & Mohamed, 2015). GAN-based approaches, particularly conditional GANs, achieved competitive fidelity for mixed-type data but showed greater sensitivity to privacy noise, corroborating observations reported in earlier studies (Goodfellow et al., 2014; Xu et al., 2019).

These findings highlight the inherent trade-off between privacy and utility, while also demonstrating that careful model selection and privacy budget allocation can mitigate fidelity degradation.

Comparison with Prior Work

Compared to earlier privacy-preserving data release methods that rely on direct perturbation or histogram-based techniques, the proposed approach offers superior scalability and expressiveness. Classical algorithms for private data release often struggle with high-dimensional data and complex dependencies, leading to significant utility loss (Hardt et al., 2012). In contrast, probabilistic generative models learn global data distributions, enabling more realistic synthetic outputs.

When compared with established systems such as PrivBayes and DataSynthesizer, the results show comparable or improved fidelity under similar privacy constraints, particularly in capturing higher-order relationships (Zhang et al., 2017; Ping et al., 2017). Furthermore, the framework aligns with recent large-scale efforts in differentially private synthetic data generation, such as those developed for the NIST competition, while offering greater flexibility in model selection and evaluation (McKenna et al., 2021).

Relative to GAN-based privacy frameworks such as PATE-GAN and DP-GAN, the proposed approach demonstrates more stable privacy-utility behavior, especially when advanced accounting mechanisms are employed (Jordon et al., 2018; Xie et al., 2018). These results are consistent with recent survey-level analyses that emphasize the importance of unified evaluation across privacy, fidelity, and attack resistance dimensions (Hu et al., 2024).

Practical Implications

Data Sharing Scenarios

The findings of this study have direct implications for privacy-sensitive data sharing across multiple domains. In healthcare, synthetic data generated with formal privacy guarantees can support clinical research, algorithm development, and cross-institutional collaboration without exposing patient-level information, addressing concerns highlighted in recent quality assessment frameworks (Vallevik et al., 2024). Similarly, in finance and government analytics, synthetic datasets can enable transparency and innovation while complying with regulatory requirements.

The demonstrated balance between privacy and fidelity suggests that probabilistic generative models can serve as a viable alternative to restricted data access

models, facilitating broader data availability for research and development.

Deployment Considerations

From a deployment perspective, the results emphasize the importance of selecting appropriate privacy budgets and model architectures based on the intended use case. Excessively strict privacy parameters can lead to unnecessary utility degradation, while overly permissive settings may undermine trust in the released data. Organizations deploying synthetic data systems must therefore align privacy configurations with regulatory standards and risk tolerance levels.

Additionally, the computational cost of training differentially private generative models should be considered, particularly for large datasets. Efficient privacy accounting and scalable training strategies are critical for real-world adoption, as highlighted in prior applied evaluations of private synthetic data systems (Rosenblatt et al., 2020).

Limitations

Scalability

Despite promising results, scalability remains a key limitation. Training probabilistic generative models with differential privacy introduces additional computational overhead due to gradient clipping, noise injection, and privacy accounting. While recent advances have improved scalability, performance can still degrade for very large datasets or frequent retraining scenarios (Abadi et al., 2016; McKenna et al., 2021).

High-Dimensional Data

Another limitation arises in high-dimensional settings. As dimensionality increases, accurately modeling complex dependencies becomes more challenging, and the impact of privacy noise is amplified. This can lead to reduced fidelity, particularly for rare categories or weak correlations. These challenges are well documented in prior work on private data synthesis and highlight the need for future research on adaptive modeling strategies and dimensionality reduction techniques (Hu et al., 2024; Vallevik et al., 2024).

Conclusion and Future Work

Summary of Findings

This study investigated the role of probabilistic generative models in synthesizing privacy-preserving big data while maintaining strong statistical fidelity guarantees. Through a systematic examination of Bayesian networks, variational autoencoders, and generative adversarial networks integrated with differential privacy mechanisms, the work demonstrated that synthetic data generation can serve as a viable alternative to direct data release in privacy-sensitive environments.

The experimental results showed that, when properly calibrated, differentially private generative models are capable of preserving key statistical properties of the original data, including marginal distributions and correlation structures, while significantly reducing the risk of privacy leakage. The analysis further highlighted the inherent trade-off between privacy protection and data utility, confirming that tighter privacy budgets lead to measurable degradation in downstream task performance. However, this degradation was not uniform across model classes, with probabilistic models exhibiting varying levels of robustness under identical privacy constraints.

Additionally, the evaluation against membership inference attacks demonstrated that incorporating formal differential privacy guarantees substantially improves resistance to adversarial exploitation compared to non-private generative approaches. These findings collectively confirm that privacy-preserving synthetic data generation, when grounded in probabilistic modeling and rigorous privacy accounting, can achieve a balanced compromise between data usability and privacy protection.

Contributions to Privacy-Preserving Data Synthesis

This research makes several important contributions to the field of privacy-preserving data synthesis.

First, it provides a unified methodological perspective that bridges probabilistic generative modeling with formal differential privacy frameworks. By jointly considering privacy guarantees, statistical fidelity, and adversarial robustness, the study moves beyond single-metric evaluations that dominate much of the existing literature.

Second, the work offers a structured comparison of multiple generative paradigms under consistent privacy constraints. This comparative analysis clarifies the strengths and limitations of Bayesian, VAE-based, and GAN-based approaches, thereby providing practical guidance for selecting appropriate models based on application requirements and risk tolerance.

Third, the study emphasizes comprehensive evaluation strategies that integrate statistical similarity

metrics, downstream utility assessments, and attack-based privacy tests. This multidimensional evaluation framework contributes to improving trust and transparency in synthetic data systems, particularly in high-stakes domains such as healthcare, finance, and public policy analytics.

Finally, the research contributes empirical evidence supporting the feasibility of deploying differentially private synthetic data as a regulatory-compliant data sharing mechanism. By demonstrating that meaningful analytical insights can be preserved without exposing sensitive individual records, the study reinforces synthetic data generation as a foundational tool for responsible data science.

Directions for Future Research

Despite its contributions, this work also reveals several promising directions for future research.

One important avenue is the scalability of privacy-preserving generative models to extremely high-dimensional and large-scale datasets. As real-world data continue to grow in complexity, future studies should investigate model architectures and optimization techniques that maintain both privacy guarantees and statistical fidelity at scale.

Another direction involves adaptive privacy budgeting strategies. Rather than allocating a fixed privacy budget uniformly across the training process, future research could explore data-dependent or task-aware budget allocation mechanisms that optimize utility while respecting global privacy constraints.

Further work is also needed to address robustness against emerging privacy attacks. While membership inference was considered in this study, future research should incorporate broader threat models, including attribute inference and model inversion attacks, particularly in adversarial deployment settings.

Additionally, integrating privacy-preserving synthetic data generation with federated and distributed learning paradigms represents a promising research frontier. Such integration could enable collaborative data analysis across institutions without centralized data sharing, thereby strengthening privacy guarantees while expanding analytical capabilities.

Finally, future studies should focus on developing standardized benchmarks and evaluation protocols for synthetic data quality and trustworthiness. Establishing widely accepted assessment frameworks would facilitate fair comparison across methods and accelerate the adoption of privacy-preserving synthetic data in practice.

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